

# Edge critical currents of dense Josephson vortex lattice in layered superconductors

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We calculate the field dependence of the critical current for the dense Josephson vortex lattice which is created by large magnetic field  $B$  applied along the layers of atomically layered superconductors. In clean samples a finite critical current appears due to the interaction of the lattice with the boundaries. The boundary induces an alternating deformation of the lattice decaying inside the sample at the typical length, which is larger than the Josephson length and increases proportional to the magnetic field. The exact shape of this deformation and the total current flowing along the surface are uniquely determined by the position of the lattice in the bulk. The total maximum Josephson current has overall  $1/B$  dependence with strong oscillations. In contrast to the well-known Fraunhofer dependence, the period of oscillations corresponds to adding *one flux quantum per two junctions*. Due to interaction with the boundaries, the flux-flow voltage for slow lattice motion also oscillates with field, in agreement with recent experiments.

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## I. INTRODUCTION

Atomically layered superconductors, such as  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x$  (BSCCO), behave as stacks of Josephson junctions. This so-called intrinsic Josephson effect has been a subject of intense research in the past decade (see reviews<sup>1,2</sup>). In spite of atomic-size separation between the neighboring junctions, in zero magnetic field the system approximately behaves like an array of independent junctions. A weak dynamic interaction between the neighboring junctions appears due to nonequilibrium effects<sup>3</sup>.

Situation is very different in the magnetic field applied along the layer direction, which generates the Josephson vortex lattice. Recently, dynamic properties of this lattice in BSCCO have been subject of extensive experimental research<sup>4,5,6,7</sup>. At high fields (above 0.5 tesla for BSCCO) the Josephson vortices homogeneously fill all layers<sup>8,9</sup> (dense Josephson vortex lattice, see Fig. 1). Josephson vortex arrays in neighboring layers have strong inductive interaction<sup>10</sup>, which strongly influences static and dynamic properties of the lattice. Due to this interaction the magnetic properties of the stack are very different from the magnetic properties of an isolated junction. A transport current flowing across the layers exerts a Lorentz force on the Josephson vortices along the layer direction. An important parameter is the *critical current* above which the lattice starts to move producing a finite voltage. This current is determined either by bulk pinning or by interaction with the boundaries. In this paper we consider the case of a homogeneous junctions and neglect bulk pinning. The simplest and most known case is a single small Josephson junction without inhomogeneities, where the field dependence of the critical current is given by the Fraunhofer dependence,  $I_c(\Phi) = I_{c0}|\sin(\pi\Phi/\Phi_0)|/(\pi\Phi/\Phi_0)$ , with  $\Phi$  being the magnetic flux through the junction. Observation of this dependence has been considered as an important confirmation of the *dc* Josephson effect. At present, the Fraun-

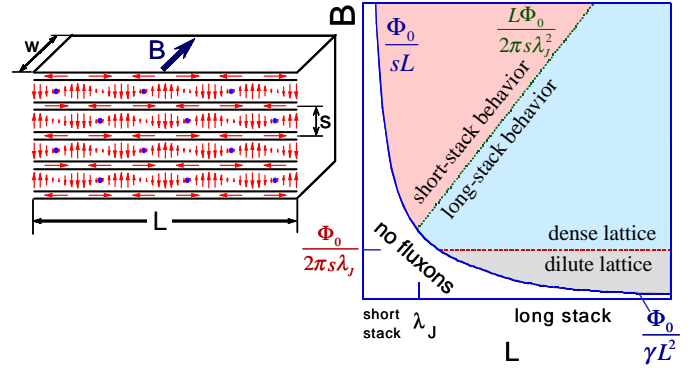


FIG. 1: *Left part*: Dense Josephson vortex lattice in a junction stack. Arrows show schematically distribution of the Josephson and in-plane currents. Points mark centers of the Josephson vortices. *Right part*: Schematic field-size diagram of a Josephson junction stack. Length and field scales are set by the Josephson length  $\lambda_J$  and the period of layered structure  $s$ . This paper is focused on the long-stack/dense lattice regime in the upper right part of the diagram.

hofer field dependence is routinely used as an indicator of the junction homogeneity. The same dependence is also expected for the junction stack with the lateral size smaller than the Josephson length<sup>11</sup>. In a single long junction the field dependence of the critical current has rather complicated structure due to multiple coexistent states of the lattice<sup>12</sup>.

In this paper we consider the critical current and lattice structure near boundaries at high magnetic fields in the regime of dense lattice in the stack of Josephson junctions. In this regime the Josephson coupling can be treated as a small perturbation, which allows to develop a full analytical description of this system. Another important simplification is that at high fields one can neglect the current self-field and neglect the difference between the magnetic field inside the junction and the external applied magnetic field.

We find that the boundary induces an alternating deformation of the lattice. Averaging out the rapid phase oscillations, we derive that the lattice deformation obeys the Sine-Gordon equation. The deformation decays inside superconductor at the typical length  $l_B$ , which is larger than the Josephson length and increases proportional to the magnetic field,  $l_B \propto B$ . Structure of the surface deformation and the total current flowing along the surface is uniquely determined by the lattice position far away from the boundary. The total current flowing through the stack is given by the sum of two independent surface currents flowing at the edges of the sample. Due to the commensurability effects, the maximum current through the stack has oscillating field dependence, which resembles the Fraunhofer dependence: it has strong oscillations and overall  $1/B$  dependence. However, the period of these oscillations corresponds to adding *one flux quantum per two junctions*.

We also calculate the flux-flow voltage for the Josephson vortex lattice slowly moving through a finite stack. Due to the interaction with the boundaries this voltage also has an oscillating contribution with period one flux quantum per two junctions. Recently, such oscillations have been observed in BSCCO mesas<sup>7</sup>, and also were seen in numerical simulations<sup>13</sup>. We find that the oscillating voltage normalized to the bare flux-flow voltage has universal dependence on the magnetic flux per one junction and the current density normalized to the Josephson current density. At high currents the relative amplitude of voltage oscillations decreases with current and field as  $1/J^2 H^2$ . We also show that in the case of dominating in-plane dissipation, typical for BSCCO, the

*absolute* amplitude of voltage oscillations weakly depends on magnetic field in a wide field range, in agreement with experiment<sup>7</sup>.

The new length scale  $l_B$  increasing with the magnetic field also sets the new field scale  $B_L$ , at which this length becomes of the order of the junction length  $L$ . Above this field the system crosses over to the regime of small junction and the field dependence of the critical current crosses over to the usual Fraunhofer dependence. Regimes of different behavior for a finite stack in magnetic field are summarized in the phase diagram shown in the right part of Fig. 1.

The paper is organized as follows. In Section II we write general relations describing the phase distribution for layered superconductor in the high magnetic field applied along the layers. In Section III we consider lattice structure near the surface, calculate the surface energy and surface current. In Section IV we calculate the oscillating field dependence of the critical current in a finite stack. In Section V we consider the field oscillations of the flux-flow voltage due to the interaction with the boundaries for slow lattice motion.

## II. GENERAL RELATIONS

Consider a stack of the Josephson junctions with length  $L \gg \lambda_J \equiv \gamma s$ , where  $\gamma$  is the anisotropy parameter, in high magnetic field  $B \gg B_{cr} = \Phi_0/2\pi\gamma s^2$  applied along the direction of the layers (see Fig. 1). At high field the screening effects can be neglected and the phase distribution  $\varphi_n(y)$  is described by the energy (per junction)

$$\mathcal{E}_J = \frac{w}{N} \sum_n \int_0^L dy \left[ \frac{J}{2} \left( \frac{d\varphi_n}{dy} \right)^2 - E_J \cos \left( \varphi_{n+1} - \varphi_n - \frac{2\pi s B}{\Phi_0} y \right) \right]. \quad (1)$$

where  $J$  is the in-plane phase stiffness,  $E_J$  is the Josephson energy per unit area, and  $w$  is the stack size in the field direction. To facilitate analysis, we introduce dimensionless parameters

$$u = y/\lambda_J, \quad \tilde{L} = L/\lambda_J, \quad e_J = \frac{\mathcal{E}_J}{E_J \lambda_J w}, \quad h = \frac{2\pi\gamma s^2 B}{\Phi_0}.$$

The reduced energy  $e_J$  is given by

$$e_J = \frac{1}{N} \sum_n \int_0^{\tilde{L}} du \left[ \frac{1}{2} \left( \frac{d\varphi_n}{du} \right)^2 - \cos(\varphi_{n+1} - \varphi_n - hu) \right]. \quad (2)$$

In a stable state the phase obeys the following equation, which expresses the current conservation,

$$\frac{d^2 \varphi_n}{du^2} + \sin(\varphi_{n+1} - \varphi_n - hu) - \sin(\varphi_n - \varphi_{n-1} - hu) = 0$$

with the boundary conditions (vanishing of the in-plane current at the boundaries)

$$\left. \frac{d\varphi_n}{du} \right|_{u=0, \tilde{L}} = 0. \quad (3)$$

In principle, a stable state can support a finite Josephson current flowing through the stack

$$j = j_J \lambda_J w \int_0^{\tilde{L}} du \sin(\varphi_{n+1} - \varphi_n - hu). \quad (4)$$

In a large-size sample this current is concentrated near the edges. In the following section we will derive a simple closed expression for the edge current.

### III. LATTICE STRUCTURE NEAR BOUNDARIES. SURFACE ENERGY AND SURFACE CURRENT.

Far from the boundaries (exact criterion will be established below) the ground state configuration corresponds to the rhombic lattice (Fig. 1) and the phase distribution can be easily calculated using expansion with respect to the Josephson current<sup>8,9</sup>

$$\varphi_n \approx n\alpha + \frac{n(n+1)}{2}\pi - \frac{2}{h^2} \sin(hu - \alpha - \pi n) \quad (5)$$

The bulk part of the energy is degenerate with respect to the phase shift  $\alpha$ . Change of  $\alpha$  corresponds to trans-

lational displacement of the lattice. This degeneracy is eliminated by the interaction with the boundaries. To study this interaction, we introduce the new phase variable  $\phi_n(u)$

$$\varphi_n = n\alpha + \pi \frac{n(n+1)}{2} + \phi_n$$

and impose the condition that  $\phi_n$  contains only oscillating contribution far away from the boundaries,  $\phi_n(u) \rightarrow -(-1)^n(2/h^2) \sin(hu - \alpha)$ . This new phase  $\phi_n(u)$  obeys the following equation

$$\frac{d^2 \phi_n}{du^2} + \sin(\phi_{n+1} - \phi_n - hu + \alpha + \pi n) + \sin(\phi_n - \phi_{n-1} - hu + \alpha + \pi n) = 0. \quad (6)$$

We assume that the boundary does not change the alternating nature of the vortex lattice and will look for solution of this equation in the form

$$\phi_n(u) = (-1)^n \phi(u).$$

For the phase  $\phi(u)$  we obtain the following equation<sup>9</sup>

$$\frac{d^2 \phi}{du^2} + 2 \cos 2\phi \sin(-hu + \alpha) = 0 \quad (7)$$

with the boundary condition  $d\phi/du|_{u=0,L} = 0$ . The energy and total Josephson current in terms of  $\phi(u)$  are given by

$$e_J(\alpha) = \int_0^{\bar{L}} du \left[ \frac{1}{2} \left( \frac{d\phi}{du} \right)^2 - \sin(2\phi) \sin(-hu + \alpha) \right], \quad (8)$$

$$\begin{aligned} j(\alpha) &= -j_J \lambda_J \int_0^{\bar{L}} du \sin(2\phi) \cos(-hu + \alpha) \\ &= j_J \lambda_J \frac{\partial e_J}{\partial \alpha} \end{aligned} \quad (9)$$

Note that the energy and current have the symmetry  $\alpha \rightarrow \alpha + \pi$ ,  $\phi(u) \rightarrow -\phi(u)$  (change of  $\alpha$  by  $\pi$  is equivalent to vertical displacement of the lattice by one junction). Therefore, the energy is  $\pi$ -periodic function of  $\alpha$ ,  $e_J(\alpha + \pi) = e_J(\alpha)$ .

We now focus on the lattice structure near the boundary  $u = 0$ . In the limit of high field  $h \gg 1$  one can separate and average out the rapidly oscillating part of the phase. This technique has been used in Ref. 9 to study melting of the Josephson vortex lattice. We split  $\phi$  into the smooth ( $v$ ) and rapidly changing ( $\tilde{\phi}$ ) components,

$$\phi = v + \tilde{\phi},$$

where the smooth component describes deformation of the lattice induced by the boundary and satisfies conditions  $dv/du \ll v$  and  $v \rightarrow 0$ , at large  $u$ . The local lattice compression is given by  $(\partial v / \partial u)/h$ . The oscillating part by definition obeys the following equation

$$\frac{d^2 \tilde{\phi}}{du^2} + 2 \cos(2v) \sin(-hu + \alpha) = 0. \quad (10)$$

Neglecting a weak coordinate dependence of  $v$  in comparison with the rapidly oscillating sine function, we obtain an approximate solution of the last equation

$$\tilde{\phi} \approx \frac{2}{h^2} \cos(2v) \sin(-hu + \alpha) \quad (11)$$

In the limit  $h \gg 1$  the oscillating phase  $\tilde{\phi}$  can be treated as a small perturbation. Subtracting Eq. (10) from Eq. (7) we obtain equation for  $v$  in the first order with respect to  $\tilde{\phi}$

$$\frac{d^2 v}{du^2} - 4\tilde{\phi} \sin(2v) \sin(-hu + \alpha) = 0$$

Substituting expression for  $\tilde{\phi}$  from Eq. (11) and averaging out rapid oscillations, we finally obtain Sine-Gordon equation for the smooth lattice deformation  $v(u)$ ,

$$\frac{h^2}{2} \frac{d^2 v}{du^2} - \sin(4v) = 0. \quad (12)$$

Using Eq. (11), we also obtain the boundary condition for  $v(u)$ ,

$$\left. \frac{dv}{du} \right|_{u=0} = - \left. \frac{d\tilde{\phi}}{du} \right|_{u=0} = \frac{2 \cos 2v_0 \cos \alpha}{h} \quad (13)$$

with  $v_0 \equiv v(0)$ . As follows from Eq. (12) the boundary deformation decays at the typical length  $l_h = h/2\sqrt{2}$  or, in the real units,

$$l_B = \gamma s \frac{\pi \gamma s^2 B}{\sqrt{2} \Phi_0}. \quad (14)$$

This important length scale appears as an interplay between the shear and compression stiffnesses of the lattice. A finite system is in the large-size limit if  $L > 2l_B$ . This condition always brakes at sufficiently large field,  $B > B_L$ ,

$$B_L = \frac{L}{\gamma s} \frac{\Phi_0}{2\pi \gamma s^2}. \quad (15)$$

Averaging out the rapid oscillations, we also derive the surface energy  $e_s(\alpha)$  and the surface current  $j_s(\alpha)$  (in units of  $j_J \lambda_J w$ ) in terms of  $v(u)$

$$\begin{aligned} e_s(\alpha) &\approx \frac{1}{h} \sin(2v_0) \cos(\alpha) \\ &+ \int_0^\infty du \left[ \frac{1}{2} \left( \frac{dv}{du} \right)^2 + \frac{1 - \cos 4v}{2h^2} \right] \\ j_s(\alpha) &= -\frac{1}{h} \sin(2v_0) \sin(\alpha) \end{aligned}$$

Equation (12) has the well-known soliton solution

$$\tan v = \tan v_0 \exp\left(-2\sqrt{2}u/h\right) \quad (16)$$

Employing the boundary condition (13), we derive relation between the boundary deformation  $v_0$  and the bulk phase shift  $\alpha$

$$\tan(2v_0) = -\sqrt{2} \cos \alpha \quad (17)$$

From this equation we can see that the maximum deformation  $v_0 = \mp \arctan(\sqrt{2})/2 \approx \mp 0.478$  occurs at  $\alpha = 0, \pi$  and the deformation vanishes at  $\alpha = \pi/2$ . Equations (16) and (17) determine the surface deformation  $v(u)$  for the given bulk phase shift  $\alpha$  (i.e., lattice position). From the surface deformation we can restore all other surface properties of the lattice. Fig. 2 shows distribution of the Josephson current near the boundary in the two neighboring junctions for two magnetic fields,  $h = 3$  and  $5$ , and different values of the  $\alpha$ . To illustrate the surface deformation, we also show the vortex center positions extrapolated from the bulk region assuming undeformed lattice.

From the obtained solution for  $v(u)$  we derive the closed analytical expressions for the surface energy and the surface Josephson current

$$e_s(\alpha) = \frac{1}{\sqrt{2}h} (1 - \sqrt{2 + \cos 2\alpha}) \quad (18)$$

$$j_s(\alpha) = \frac{1}{\sqrt{2}h} \frac{\sin 2\alpha}{\sqrt{2 + \cos 2\alpha}} \quad (19)$$

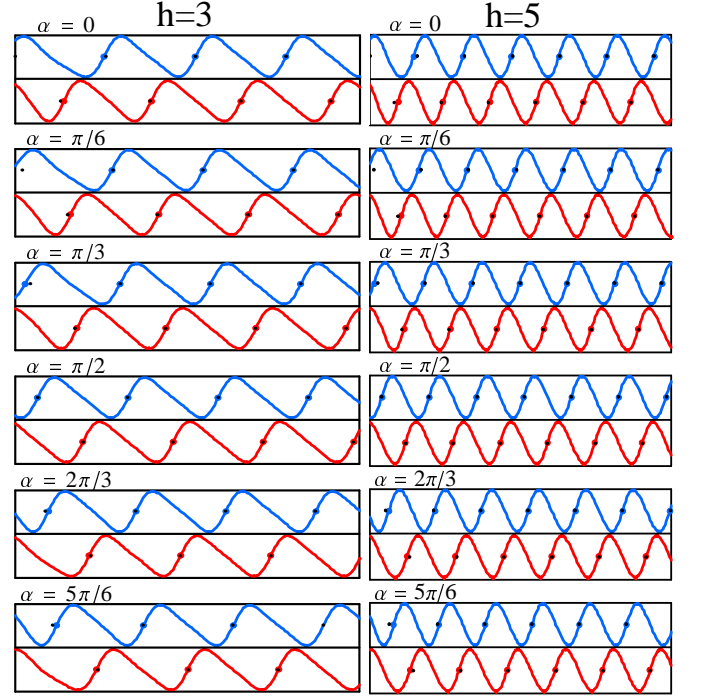


FIG. 2: Distribution of the Josephson current near the boundary  $u = 0$  in the two neighboring junctions for two magnetic fields,  $h = 3$  and  $5$ , and different values of the bulk phase shift  $\alpha$ . Small black points mark the positions of the vortex centers extrapolated from the bulk region assuming undeformed lattice. The real vortex positions deviate from these marks due to the surface deformation of the lattice.

The  $\alpha$ -dependence of the surface current is plotted in the left panel of Fig. 3. The maximum surface current,  $j_{s,\max}$ , is achieved at  $\cos 2\alpha = -2 + \sqrt{3}$  and in real units is given by

$$j_{s,\max} = \sqrt{2 - \sqrt{3}} j_J \frac{\Phi_0}{2\pi s B}. \quad (20)$$

Two possible directions of this current correspond to the Lorentz force on Josephson vortices directed from the boundary and towards the boundary, promoting either entrance or exit of vortices. In contrast to the regime of dilute lattice at  $B < B_{cr}$ , in which the maximum values of the entrance and exit current are very different (see, e.g., Ref. 14), in the dense lattice they are approximately the same. It is interesting to note that, up to numerical factor, the expression for the entrance current in the dilute lattice regime coincides with Eq. (20).

#### IV. CRITICAL CURRENT OF A FINITE STACK

Consider now the total current flowing through a finite stack. For the given phase shift  $\alpha$  (lattice position) the total current,  $J_L(\alpha) \equiv J_L(\alpha, hL, h)$ , is the sum of the two surface contributions coming from the boundaries  $u = 0$

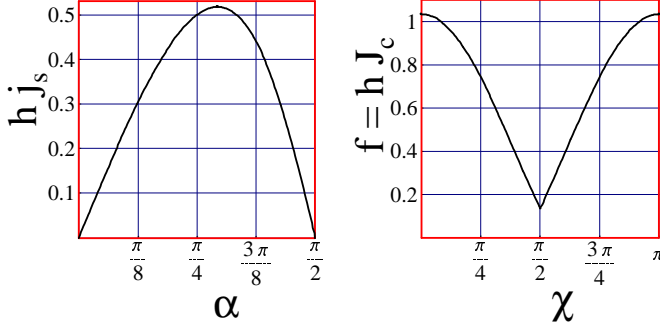


FIG. 3: *Left Panel:* Plot of the function  $h j_s = \frac{1}{\sqrt{2}} \frac{\sin 2\alpha}{\sqrt{2+\cos 2\alpha}}$ , which gives dependence of the surface current on the phase shift  $\alpha$  (see Eq. (19)). *Right Panel:* Dependence of function  $f = h J_c$ , which determines the shape of the critical current oscillations in stack of long junctions, on the reduced flux through one junction  $\chi = 2\pi\Phi/\Phi_0$  in Eq. (23).

and  $u = L$ ,

$$J_L(\alpha) = \frac{1}{\sqrt{2}h} \left( \frac{\sin 2\alpha}{\sqrt{2+\cos 2\alpha}} - \frac{\sin 2(hL-\alpha)}{\sqrt{2+\cos 2(hL-\alpha)}} \right) \quad (21)$$

Introducing notation  $\chi = hL = 2\pi\Phi/\Phi_0$ , where  $\Phi = BsL$  is the magnetic flux through one junction, we obtain that the maximum current  $J_c(\chi, h) = f(\chi)/h$  is given by

$$J_c(\chi, h) = \max_{\alpha} [J_L(\alpha, \chi, h)] \quad (22)$$

Numerically obtained dependence of  $f = h J_c$  vs  $\chi$  is plotted in the right panel of Fig. 3. In the real units, the field dependence of the maximum Josephson current in the field range  $\Phi_0/2\pi\gamma s^2 < B < L\Phi_0/\pi\gamma^2 s^3$  is given by

$$I_c(B) = I_J \frac{\Phi_0}{2\pi s L B} f\left(\frac{2\pi s L B}{\Phi_0}\right), \quad (23)$$

where  $I_J = j_J L w$  is the maximum Josephson current through the stack at zero field, and oscillating function  $f(\chi)$  is plotted in the right panel of Fig. 3. To facilitate comparison with experiment we also obtain an interpolation formula for  $f(\chi)$  for  $0 < \chi < \pi/2$ ,  $f(\chi) \approx 0.128 + 0.888 \cos(\chi) + 0.021 \cos(3\chi)$ . The derived field dependence of the critical current somewhat resembles the well-known Fraunhofer dependence in a single small-size Josephson junction: it has overall  $1/B$  dependence with large oscillations. However, it has a very different physical origin and also has several qualitative differences. The most important difference is that the period of oscillations corresponds to adding *one flux quantum per two junctions*, i.e., it is two times smaller than for the Fraunhofer oscillations. This kind of oscillations have been recently observed in the flux-flow voltage of BSCCO mesas<sup>7</sup>. Also, contrary to the Fraunhofer dependence, the points where the magnetic flux inside the junction equals to integer flux quanta, correspond to

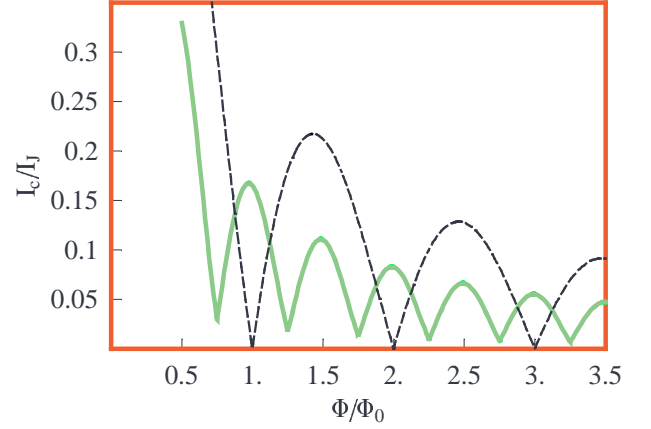


FIG. 4: Thick solid line shows dependence of the critical current for the stack of long Josephson junctions on the magnetic flux per junction. For comparison we also show by dashed line the Fraunhofer dependence  $|\sin(\pi\Phi/\Phi_0)|/(\pi\Phi/\Phi_0)$  which describes the field dependence of the critical current in a single small Josephson junctions.

the local maxima of the critical current. Dependence of the critical current on the magnetic flux per junction is plotted in Fig. 4. For comparison, the Fraunhofer dependence is also shown. The dependence (23) holds until two edges give independent contributions to the total current. This condition breaks when the magnetic field exceeds  $B_L = L\Phi_0/(2\pi\gamma^2 s^3)$ . At higher fields the field dependence crosses over to the usual Fraunhofer dependence. Recently, this crossover has been studied by numerical simulations<sup>13</sup>.

## V. OSCILLATING FLUX-FLOW VOLTAGE

When the external current exceeds the critical current, the lattice starts to move. At slow motion the surface deformation has time to adjust to the current lattice position. In this case the surface energy produces a periodic potential for the moving lattice and one can use static results to predict the I-V dependencies. Time variation of the lattice phase shift obeys equation

$$\nu_{ff} \frac{d\alpha}{dt} + J_L(\alpha) = J, \quad (24)$$

where the total surface current  $J_L(\alpha) \equiv J_L(\alpha, hL, h)$  is given by Eq. (21) (for brevity we again skip in equations its dependence on the magnetic field and size) and the viscosity coefficient,  $\nu_{ff}$ , is related to the flux-flow resistance of the stack,  $R_{ff}$ ,

$$\nu_{ff} = \frac{N\Phi_0}{2\pi c R_{ff}}.$$

where  $N$  is the number of junctions in the stack. Voltage drop per one junction  $U$  is related to  $d\alpha/dt$  by the

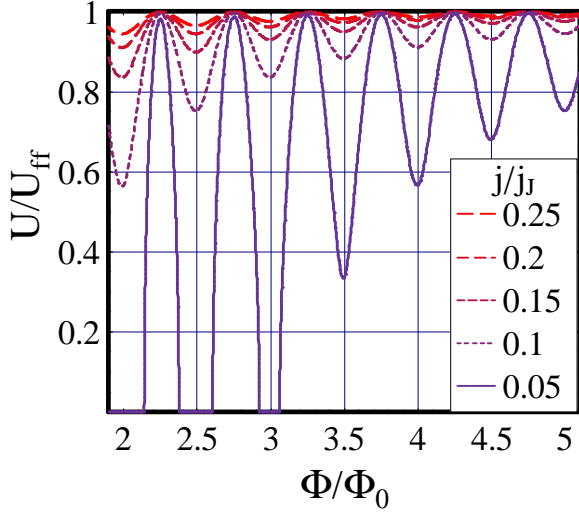


FIG. 5: Oscillating dependencies of the reduced voltage on the magnetic flux per one junction for different current densities. The plot represents a universal behavior for junction stacks with different sizes.

Josephson relation

$$U = \frac{\Phi_0}{2\pi c} \frac{d\alpha}{dt}.$$

Solution of Eq. (24) is given by the implicit relation

$$\int_0^\alpha \frac{\nu_{ff} d\alpha'}{J - J_L(\alpha')} = t,$$

from which we obtain the average phase change rate

$$\frac{d\alpha}{dt} = \left[ \frac{1}{\pi} \int_0^\pi \frac{\nu_{ff} d\alpha}{J - J_L(\alpha)} \right]^{-1}$$

and the flux-flow voltage

$$\frac{U}{U_{ff}} = \left[ \frac{J}{\pi} \int_0^\pi \frac{d\alpha}{J - J_L(\alpha)} \right]^{-1} \quad (25)$$

with  $U_{ff} = R_{ff}J$  being the bare flux-flow voltage without periodic potential. Because the surface current  $J_L(\alpha) \equiv J_L(\alpha, hL, h)$  oscillates with the magnetic field, this flux-flow voltage will also experience similar field oscillations. Such oscillations have been recently observed by Ooi *et al.*<sup>7</sup> Using scaling property of the current  $J_L(\alpha, hL, h) = LF(\alpha, hL)$  we can see from Eq. (25) that the reduced flux-flow voltage  $U/U_{ff}$  depends only on two parameters: the current density normalized to the Josephson current density,  $i \equiv J/L \equiv j/j_J$ , and the magnetic flux through one junction,  $\Phi = BsL$ . This allows for a unified description of the voltage oscillations in junction stacks with different sizes. Oscillating dependencies of  $U/U_{ff}$  on the magnetic flux  $\Phi$  for different current densities are shown in Fig. 5.

At large currents one can expand this result with respect to surface current and obtain a small correction to

the flux-flow voltage

$$\frac{U}{U_{ff}} \approx 1 - \frac{g(0) + g(\chi)}{\pi J^2 h^2} \quad (26)$$

where the oscillating part is described by dimensionless function

$$g(\chi) = \int_0^\pi \frac{\sin 2\alpha \sin 2(\alpha - \chi)}{\sqrt{2 + \cos 2\alpha} \sqrt{2 + \cos 2(\chi - \alpha)}} \frac{d\alpha}{\pi}, \\ \approx 0.263 \cos 2\chi + 0.0046 \cos 4\chi.$$

Therefore, the absolute amplitude of voltage oscillations is given by

$$\delta U = 0.527 \frac{R_{ff}}{J} (j_J \lambda_J)^2 \left( \frac{\Phi_0}{2\pi s \lambda_J B} \right)^2. \quad (27)$$

Independently from the dissipation mechanism, the relative amplitude of oscillations  $\delta U/R_{ff}J$  decreases with field as  $1/B^2$ . In the regime of dominating in-plane dissipation, typical for BSCCO, the flux-flow resistance is given by<sup>15</sup>

$$R_{ff} = R_c \frac{B^2}{B^2 + B_\sigma^2}; \quad B_\sigma = \sqrt{\frac{\sigma_{ab}}{\sigma_c}} \frac{\Phi_0}{\sqrt{2}\pi\gamma^2 s^2}$$

where  $\sigma_{ab}$  and  $\sigma_c$  are the components of quasiparticle conductivity. Using this relation one can rewrite Eq. (27) as

$$\delta U = \frac{R_{ab} J_M^2}{2J} \frac{1}{1 + B^2/B_\sigma^2} \quad (28)$$

with  $J_M = c\Phi_0/8\pi^2\lambda_{ab}^2$ ,  $R_{ab} = Ns/S\sigma_{ab}$ , and  $S = wL$  is the junction area. For BSCCO, typically,  $B_\sigma \sim 10T$ . In the region  $B < B_\sigma$  the oscillation amplitude weakly depends on the magnetic field, in agreement with experiment (see Fig. 2 in Ref. 7). It is interesting to note that in this regime it is mainly determined by the in-plane parameters of superconductor.

## VI. SUMMARY AND ACKNOWLEDGEMENTS

In conclusions, we found that the edge current of the dense Josephson lattice is uniquely determined by the magnetic field and position of the lattice in the bulk. Near the surface the lattice has alternating deformation, which decays inside superconductor at the typical length which is proportional to the magnetic field. Due to the rhombic lattice structure, both the critical current and the flux-flow voltage at small velocities have oscillating field dependencies with the period of one flux quantum per two junctions up to the size-dependent magnetic field.

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